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# To Each According to its Degree: The Meritocracy and Topocracy of Embedded Markets

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**A system is said to be meritocratic if the compensation and power available to individuals is determined by their abilities and merits. A system is topocratic if the compensation and power available to an individual is determined primarily by her position in a network. Here we introduce a model that is perfectly meritocratic for fully connected networks but that becomes topocratic for sparse networks-like the ones in society. In the model, individuals produce and sell content, but also distribute the content produced by others when they belong to the shortest path connecting a buyer and a seller. The production and distribution of content defines two channels of compensation: a meritocratic channel, where individuals are compensated for the content they produce, and a topocratic channel, where individual compensation is based on the number of shortest paths that go through them in the network. We solve the model analytically and show that the distribution of payoffs is meritocratic only if the average degree of the nodes is larger than a root of the total number of nodes. We conclude that, in the light of this model, the sparsity and structure of networks represents a fundamental constraint to the meritocracy of societies.**

In the ideal world of Arrow-Debreu, every transaction that creates a surplus takes place. Unfortunately, we don't live in that world. An important difference between our world and that of Arrow-Debreu is that, in our world, every pair of individuals is not connected directly, but indirectly via networks of intermediaries, agents and middleman who expect to benefit from their intermediating role.

As Granovetter[1] pointed out more than two decades ago, our economy is *embedded* in social networks. These are networks that beget commercial interactions, and that are begot by them. For Granovetter, the cultivation of personal relationships between traders and customers assumes an equal or higher importance than the economic transactions involved. Economic exchanges are not carried out exclusively among strangers, but often incorporate individuals involved in long-term continuing relationships.

The embeddedness of markets is particularly important when links are costly. If links were costless society would behave similar to a fully connected network, and we will be back to the idealized world of Arrow-Debreu. When links are costly, however, embeddedness becomes extreme and markets are restricted by the structure of the social networks that co-exist with them.

In this paper, we explore the redistributive consequences of the networks underlying economic activity by introducing a model with tunable embeddedness. The model separates the income of agents into two sources, the income obtained from the content agents produce, and the income that agents obtain from their intermediation role. We solve the model analytically and show that as networks become sparser, the model transitions from the meritocratic regime of Arrow-Debreu to what we call a *topocratic* regime, where the position of an individual in a network becomes the most important factor determining the compensation it receives.

Understanding the redistributive consequences of networks is important in a world where markets are composed of a mix of socially embedded links and commercial "arm-length" relationships[2,3]. Yet, even in a world where arm-length relationships are dominant, the assumption of fully connected networks is too hopeful. Possible transactions might not take place because individuals are uncertain about the quality of the goods being offered[4–6] or due to search frictions[7,8]. These market failures are partially compensated by the emergence of middlemen who are experts at reducing information asymmetries and search frictions, but who also act as hubs controlling information flows in the network. As Ronald Burt points out, the position that a middleman occupies in the network is a source of advantage, as intermediating positions constitute part of what he has termed the "social capital of structural holes"[9,10].

In recent decades the social and economic role of networks has received an increased level of recognition. Economists have modeled the networks that emerge from strategic interactions[11–14], as well as the inequality in the distribution of payoffs expected in these equilibrium networks[15–17]. Our model contributes to this literature by separating the content producing role of an agent from its role as an intermediary. This separation allows us to study the conditions under which the payoffs received by an individual are determined by the content she produces, or by her position in a social or professional network. To distinguish between these two payoff distribution regimes, we label the outcome of the system as meritocratic, when the distribution of payoffs is determined primarily by an agent's ability to produce quality content, and *topocratic* when the distribution of an agent's payoffs is determined primarily by her position in the network. We find that the transition to topocracy is mediated by network density, with topocracy becoming the dominant regime of sparse networks. In general, we find that the critical connectivity required to transition from topocracy to meritocracy goes as a root of the size of the network ($N^a$), with $a < 1$. This non-linear relationship means that the transition point is highly sensitive to both, the structure of the network and the algorithm used to distribute payoffs among individuals.

The implications of a root-rule of this kind can be explained by looking at numerical examples. Consider a network with as many nodes as people in the United States ($N = 3 \times 10^8$). In this case an $N^{1/2}$ rule implies that meritocracy kicks in for connectivities above 17,320 links per node. This is certainly too large, meaning that an $N^{1/2}$ rule would imply that the U.S. is topocratic. An $N^{1/4}$ rule, on the other hand, implies a minimum average connectivity of only 131 links per node, which represents a reasonable number of social connections[18]. Hence, when the transition from meritocracy to topocracy is mediated by an $N^{1/4}$ rule the implications for the U.S. would be that this is likely to be meritocratic.

The fact that in our model the transition between meritocracy and topocracy depends predominantly on the density of the network has two important implications. The first one is that the strong dependence of meritocracy on density makes the results of the model robust to different network formation mechanisms. Here, we can separate between two possibilities. First, there is the world in which the connectivity of individuals is determined largely by processes that are exogenous to them. This is a world where connectivity begets connectivity, such as in the case of the Barabási Albert Model[19], the Yule Process[20], the Price Model[21], Merton's Cumulative Advantage (or Matthew effect)[22], or Herbert Simon's modified version of the Yule Process[23]. The second possibility is one in which the position that an individual occupies in a network is determined endogenously, for instance through strategic interactions[11–14]. Yet, when the density of the network is bounded–due for instance to the high cost of links–differences between link formation mechanisms, whether endogenous or not, should not introduce substantial changes to the meritocratic properties of the system. In other words, when the density of the network is the main feature determining whether the markets embedded in them are meritocratic or not, the forces of endogenous network formation will only be able to modify this outcome slightly (we note that this is not true for an endogenous network formation process with full information. Yet, assuming full information in the network formation is equivalent to assuming a fully connected network).

The second implication that we would like to highlight is that the model predicts that meritocracy increases in societies that become better connected. This is an important implication given current changes in technology. Recent changes in communication technologies have increased the connectivity of our society, by reducing the cost of both social and commercial interactions. Most studies have emphasized the role of communication technologies on social participation and collaboration. Our results suggest that this technological change might also have important long term effects on the meritocracy of economies. Content producers, whether these are musicians or artists, can now market their content directly to a large number of individuals, even though this causes an information overload[24,25] that puts us far from the idealized limit of fully connected networks. Nevertheless, in the light of this model, changes in communication technology should increase the meritocracy of markets–when holding population size constant. So the good news is that recent changes in technology should help make our society more meritocratic.

## Results

**Modelling a networked market.** Consider a world where individuals produce, distribute and consume content. The content produced by individuals can be abstracted as widgets of low marginal production cost, or cultural goods, such as books, films or music. To simplify the discussion we label the content producing role of an individual as her *Rockstar* role, since the payoffs collected via this role depend on the popularity of the content she produces. We call the intermediation role of an individual as her *Middleman* role, since the payoffs collected via this role are proportional to the transactions that she helps complete. The model is fully specified by three sets of assumptions:

1. **Initial conditions**: The model begins with an exogenously determined network in which each node represents an individual. Each individual is endowed with a single parameter $T$ representing its *Talent*, or ability to contribute to society. The talent $T$ is fixed for the duration of the model and determines the fraction of individuals that are willing to purchase the content produced by an individual. For example, an individual with $T = 0.3$ will sell its cultural good to 30% of all other individuals in the network. For simplicity, we draw $T$ from a uniform distribution ($U \sim [0, 1]$).
2. **Value generation**: At each time step each node produces a new good (i.e. song, book, article, movie, etc.). These are non-rival goods, meaning that copies can be made at no cost. The goods made by an individual in her *Rockstar* role are purchased by a fraction $T$ of all individuals. For simplicity, we choose the price to be constant and equal for all purchases. We later show that our results do not depend on prices.
3. **Value distribution (a.k.a. payoff structure)**: If a *Rockstar* is not connected directly to an individual willing to purchase her content, the purchase is completed through the shortest path. In this case, the total revenue of the sale is distributed equally between each of the individuals in the path. For example, a purchase completed through a path of length three will give 1/3 of the payoff to the individual producing the content (the *Rockstar*), and to each intermediary (the *Middlemen*). Later, we generalize the model to other profit sharing rules.

The assumptions and the model are explained diagrammatically in figure 1 A.

Assumptions (1) to (3) define a model in which individuals collect payoffs by either producing content, or by being intermediaries in the distribution of the content produced and consumed by others. Hence, individuals have two sources of income: one that depends on talent, which we call meritocratic, and one that depends on their position in the network, which we call topocratic. We note that the topocratic channel defined by an individual's *Middleman* role is useful for the system, since without it transactions would not be completed. We note that the market is completed only as long as there are indirect paths between every pair of nodes.

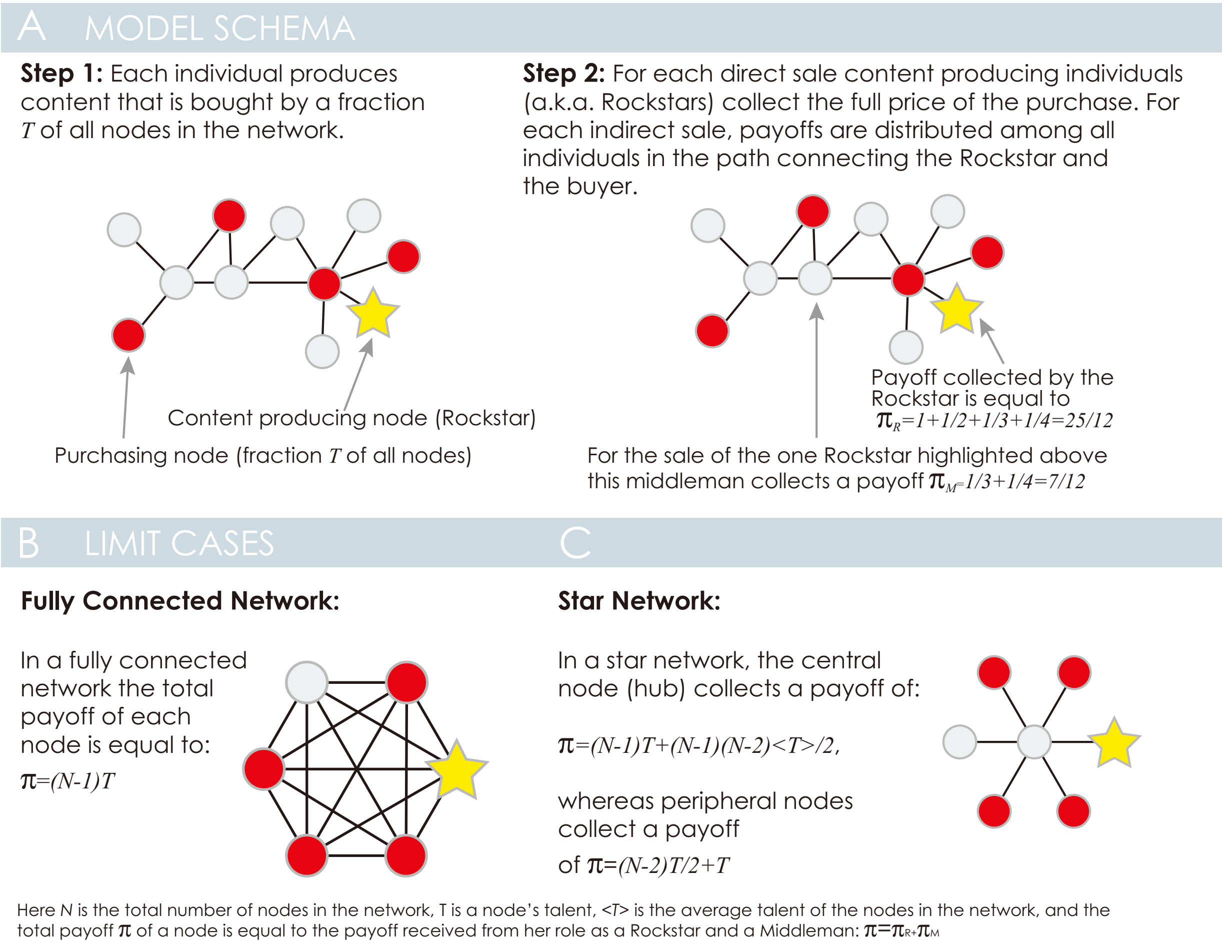


**Figure 1** | (A) Schematic representation of model. (B) Solution for a fully connected network. (C) Solution for a star-network.

**Meritocratic and topocratic regimes - limit cases.** In a fully connected network, the model has a trivial solution where payoffs come solely from the production of content, since intermediation is not necessary, and hence, avoided. In this case, the payoffs ($\pi_i$) follow Talent ($T_i$) exactly, since an individual with talent $T_i$ receive a payoff $\pi_i = T_i(N - 1)$, where $N$ is the number of nodes in the network (Figure 1 B). Hence, when the network is fully connected, the model describes a system that is perfectly meritocratic, i.e., payoffs are perfectly correlated with talent.

In a world where the network is not fully connected the income of an individual will depend not just on its talent, but also on the betweenness centrality of an individual (the betweenness centrality of a node is the number of shortest paths going through that node). Here, the extreme case is a star-network with one central node, or hub (Figure 1 C). In this case, a node in the periphery of the star network receives an income equal to:

$$\pi_i = (N-2)T_i/2 + T_i \tag{1}$$

On the other hand, the hub receives a payoff equal to

$$\pi_i = \langle T \rangle (N-1)(N-2)/2 + T_i(N-1) \tag{2}$$

where $\langle T \rangle$ is the average talent of the system, the first term represents the revenue intermediated by the hub, and the second term represents the probability that the hub is a direct buyer of the peripheral node's content.

We note that the maximum possible payoff that can be obtained from intermediation is $\sim N^2$, whereas the maximum possible payoff that can be obtained from producing content is $\sim N$. This large difference emerges because the payoff that an individual gets from its *Middleman* role grows with the number of possible links in the system, which is quadratic on the number of nodes $N$, whereas the maximum income for the *Rockstar* role is bounded by the number of nodes in the network ($N$). This difference does not depend on our choice of distribution of talents, or the way in which revenue is distributed along the chain, so it is a fundamental difference between the intermediation role of *Middlemen* and the production role of *Rockstars*. It therefore represents a fundamental constraint to any model considering this duality of behaviors.

**Meritocratic and topocratic regimes - general case.** *Determinants of individual payoffs*. We begin by splitting the payoff collected by an individual through each of her two roles

$$\pi_i = \pi_{R_i} + \pi_{M_i} \tag{3}$$

where $\pi_{R_i}$ and $\pi_{M_i}$ indicate, respectively, the payoffs from the *Rockstar* and *Middleman* behavior.

In an Erdös-Rényi network (ER network)[26] the degree of a node is well approximated by the average degree of the network. Hence, we can approximate the payoff that an individual gets from her *Rockstar* role by her talent times the number of individuals at distance $d$ from her discounted by the length of the chain connecting her to each individual. Hence, in a random network where the average connectivity or degree is equal to $\langle k \rangle$, $\pi_R$ can be approximated by:

$$\pi_{R_i} = T_i \left[ \langle k \rangle + \frac{\langle k \rangle^2}{2} + \frac{\langle k \rangle^3}{3} + \ldots + \frac{\langle k \rangle^\ell}{\ell} \right] = T_i \sum_{j=1}^{\ell} \frac{\langle k \rangle^j}{j}, \tag{4}$$

where the cutoff $\ell$ is equal to the average path length of the network, which in a random network is well approximated by:

$$\ell \simeq \frac{\ln N}{\ln\langle k\rangle} \tag{5}$$

The income that individuals earn from their behavior as *Middlemen*, $\pi_M$ can be obtained by noticing that in every non-direct sale conducted through a chain of length $d$, the $d - 1$ *middlemen* participating get an equal share of the purchase. In this way, the total income in the network that is collected by *Middlemen* can be written similarly to (4), as

$$\Pi_M = \sum_j \pi_{M_j} = N\langle T\rangle\left[\frac{\langle k\rangle^2}{2} + \frac{2\langle k\rangle^3}{3} + \frac{3\langle k\rangle^4}{4} + \ldots + \frac{(\ell-1)\langle k\rangle^\ell}{\ell}\right] = N\langle T\rangle\sum_{j=2}^{\ell}\frac{j-1}{j}\langle k\rangle^j. \tag{6}$$

The payoff collected by a single *middleman* can be obtained by taking the share of shortest paths going through an agent with degree $k_i$. In a random network the number of shortest paths going through a node is given by $k_i^2 \Big/ \sum_j k_j^2$[27]. Hence, the average payoff collected by a *Middleman* is:

$$\pi_{M_i} = N\langle T\rangle\frac{k_i^2}{\sum_\upsilon k_\upsilon^2}\sum_{j=2}^{\ell}\frac{j-1}{j}\langle k\rangle^j = k_i^2\frac{\langle T\rangle}{\langle k\rangle^2+\langle k\rangle}\sum_{j=2}^{\ell}\frac{j-1}{j}\langle k\rangle^j. \tag{7}$$

Finally, we can use equations (4) and (7) to write down a general formula for the payoff $\pi$ of individual $i$ as a function of both, its talent $T_i$ and its connectivity $k_i$.

This formula takes the general form:

$$\pi_i = CT_i + Bk_i^2 \tag{8}$$

where $C$ and $B$ are given by

$$C = \sum_{j=1}^{\ell}\frac{\langle k\rangle^j}{j} \text{ and } B = \frac{\langle T\rangle}{\langle k\rangle^2+\langle k\rangle}\sum_{j=2}^{\ell}\frac{j-1}{j}\langle k\rangle^j.$$

We note we have assumed all nodes to belong to the network's giant component–there are no isolated nodes or clusters in the network. For networks made of several components our results are still valid by considering $N$ to be the number of nodes in the component in question.

For sparse networks, as Figure 2 illustrates, high payoff individuals concentrate on the core of the network irrespective of their talent. Conversely, when the network becomes denser, the opposite becomes true: talented individuals, irrespective of their position in the network, are the ones collecting the highest payoffs.

*Transition threshold.* Since the model is perfectly meritocratic for a fully connected network, and highly topocratic for a star network, the natural question to ask is when does the transition between meritocracy and topocracy takes place. We define the regime as meritocratic when the fraction of payoffs paid for the creation of content is larger than the fraction of payoffs paid for their distribution (see Methods for an alternative definition based on the correlation between talent and payoffs). Formally, we can express this as:

$$\frac{\Pi_R}{\Pi} > \frac{1}{2}$$

where $\Pi_R$ corresponds to aggregate payoffs associated with the role of *Rockstars* and $\Pi$ corresponds to aggregate total payoffs. For an ER network, the regime will be meritocratic if the following condition holds for the relationship between average connectivity and network size (see Methods for derivation):

$$\langle k\rangle > N^{1/2} \tag{9}$$

Figure 3 shows how the wealth generated in the model is distributed between the two possible activities, *Rockstars* and *Middlemen*, as a function of the average network connectivity. The transition point from topocracy to meritocracy ($\langle k\rangle = N^{1/2}$) is indicated with a dashed vertical line. We note that for small values of $\langle k\rangle$ the total payoff of the system decreases, since the network becomes fragmented and there are transactions that are not completed.

Finally, we note that there is also a structural interpretation for the $\langle k\rangle = N^{1/2}$ threshold. When the average connectivity of the network is equal to the square root of the total number of nodes, the average distance in the network is two, meaning that individuals are no further than two hops away. Hence, the $\langle k\rangle = N^{1/2}$ rule obtained in this case is equivalent to saying that topocracy emerges in random networks when the average path length is larger than two, and hence, that six-degrees of separation imply a highly topocratic system.

**Alternative sharing rule: comissions.** Next, we extend the model to a payoff sharing rule in which *Middlemen* get a percentage of the total transactions in which they participate. For instance, they get a 10 percent commission of the transactions in which they are directly involved. We note that this is not the same as a ten percent commission of the entire purchase. For example, in a purchase of an item of price one completed by a chain of length three, the first

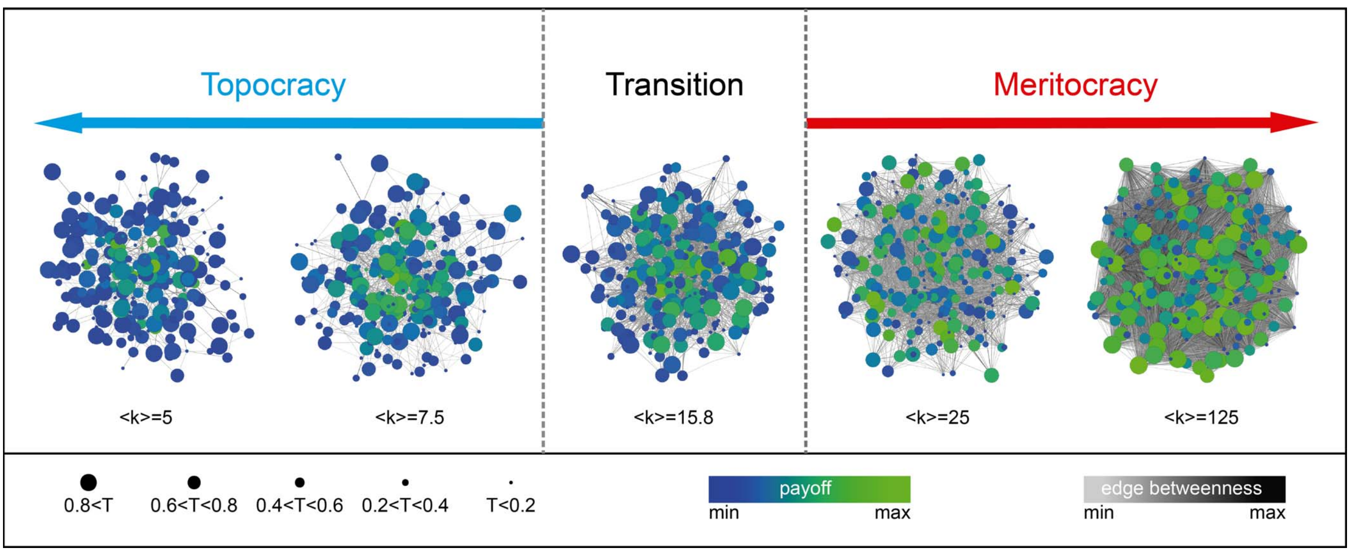


**Figure 2 | Payoff distribution for different levels of average connectivity on a network of size $N = 250$.**

*middlemen* gets 0.1, the second gets $(1 - 0.1) \times 0.1 = 0.09$ and the *Rockstars* gets $1 - 0.1 - 0.09 = 0.81$.

With these assumptions, the payoff of a *Rockstar* is given by

$$\pi_{R_i} = T \sum_{i=1}^{\ell} \langle k \rangle^i (1-\alpha)^{i-1}, \tag{10}$$

which yields the following conditions for the regime to be meritocratic (see Methods for derivation):

$$\langle k \rangle > N^{\frac{\ln(1-\alpha)}{\ln(1-\alpha)/2}} \tag{11}$$

Figure 4 shows the share of total payoffs collected by *Rockstars* and *Middlemen* as a fraction of the average connectivity ($\langle k \rangle$) and the percentage collected by each *middlemen* ($\alpha$). When $\alpha$ is small, the transition to meritocracy takes place at low connectivities. For instance, when $\alpha = 0.1$ the transition to meritocracy takes place at $\langle k \rangle = N^{0.13}$. For a network with $N = 10^7$ nodes this implies a threshold connectivity of just $\langle k \rangle = 8$, meaning that very little connectivity is required for the system to become meritocratic. For large percentages, however, the transition to meritocracy is not that easy. When $\alpha = 1/2$ the transition is once again at $\langle k \rangle = N^{1/2}$. The $N^{1/2}$ rule might seem counterintuitive when considering a percentage rule, since in a percentage rule a 50% commission would imply an extremely fast decay on the remainder received by the *Rockstar*. Yet, when $\langle k \rangle = N^{1/2}$ all individuals are on average, no more than two hops away from each other, and hence the payoffs are not distributed via long chains.

## Discussion

For inequality to exist, there must be a story justifying why those at the top are entitled to more than those at the bottom. Centuries ago, European monarchs used divinity to justify their privileged positions. It was their connection to God what made them special, and, ultimately legitimized their special status[28]. In our modern era, justifying inequality based on divine right is no longer acceptable and a number of scientific dictums have emerged to fill the societal role once filled by holy explanations. Marx and Friedman pronounced themselves in this area, and although they did not share their view on economics, they shared the sense of poetry in their expression. In The Critique of the Gotha Program, Marx famously said "From each according to his ability, to each according to his need"[29]. Through this phrase Marx attempted to convey what he thought should be the economic relationship between an individual and society: individual's contribute according to their ability, but should receive according to their need. More than 100 years later Friedman used Marx phrasing to voice what he thought was the right interpretation of this relationship "to each according to what he and the instruments he owns produce"[30].

Economies, however are made of more than talents and property. As the social capital and embeddedness theory has often remarked[1–3,9,10,31], people are structured in social networks. These networks can be instrumental drivers of inequality in centrally planned economies, but also in free markets, where connections to business elites can take the role that connections to party leaders have in autocratic regimes. Networks, thus, affect the functioning of decentralized economies and limit the often desired equality of opportunity since they help determine the information and resources available to each individual. In the context of equality of opportunities, John Rawls argues that equality of fair opportunity will only be satisfied in a society where the same native talent and the same ambition have the same prospects of success[32,33]. Policy-makers who adhere to Rawles ideas have emphasized the field-leveling role of inheritance taxes, education and anti-discriminatory policies in the labor market. Yet, opportunities are not constrained only by talents, education and property, but also by the connections available to each individual, which cannot be taxed. Hence a thorough understanding of the meritocracy of market mechanisms cannot be achieved without understanding the effects of an individual's position in a network and its relative effect with respect to other forms of advantage where field leveling policies do exist.

In a 21st century context the results of this paper also speak about the social changes that are implied by recent changes in technology. In recent years the emergence of the internet has given rise to a world in which it is much easier for individuals to market directly to each other, or at least, through one large intermediator (such as iTunes, Amazon or eBay). Our model predicts that these changes should increase the meritocracy of society since they help reduce the long chain of intermediations that consume valuable payoffs in a poorly connected society.

But does this mean that denser networks are unambiguously preferable to sparser networks? Not quite. Making such a judgement would require weighing the effects that network density has on meritocracy with its effect on other social and economic outcomes. Social networks do not only affect the distribution of payoffs among content producers and middlemen, but also are known to affect the outcome of coordinated collective action. For instance, evolutionary

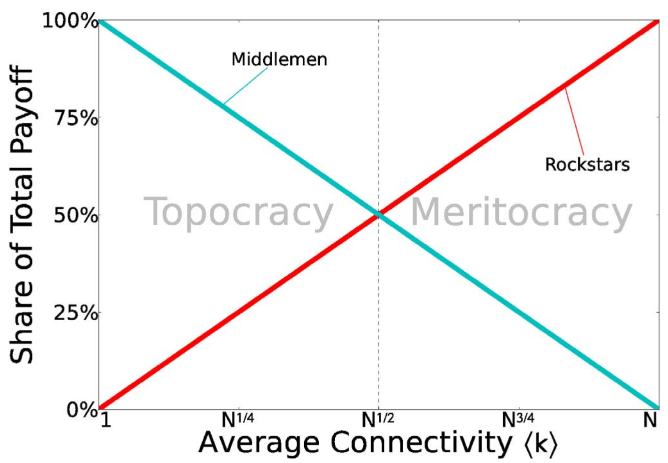


**Figure 3 | Share of total payoffs (eq. 13) as a function of the average network connectivity for a random Erdös-Rényi network and a proportional payoff sharing rule.**

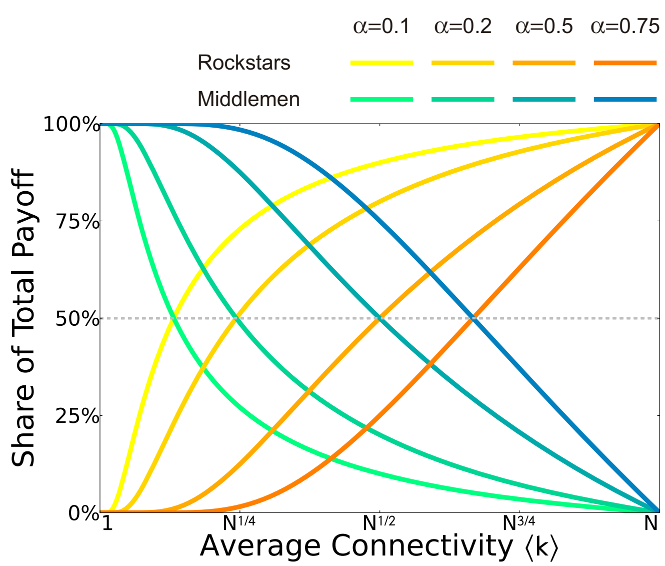


**Figure 4 | Share of total payoffs as a function of average network connectivity for a random Erdös-Rényi network and a sequential payoff sharing rule according to equation(18).**

game theory suggests that cooperative strategies are more likely to emerge in networks that are not highly connected. In a public good game sparse network prevents free-riders from prospering because free-riders cannot sustain enough links to exploit multiple neighbors[34–36]. Thus, in a sparse network, the same agents that benefit from their position as middleman might be the same agents that play a crucial role enhancing cooperation. Therefore, making a judgement on whether a denser or sparser network is more beneficial for society in general, is a matter that cannot be answered easily, since it requires weighing the effects of the network structure on meritocracy and cooperation, but also, on other relevant outcomes, from the preservation of cultural diversity to the spread of disease.

Finally, the model also invites us to explore a number of different generalizations. Two generalizations seem particularly interesting. First, is the development of an endogenous model in which individuals can invest in the creation of new links, or could modify their talent, for instance, by investing in education. The ability of such a dynamic process to restore the meritocracy of the system, will be limited whenever the maximum connectivity of nodes is bounded. This is likely to be true due to time constraints and the limited cognitive capacities of individuals, but it would nevertheless be interesting to explore the strategies that can help balance meritocracy in a limited setting. The other generalization involves the use of a model of this kind to explain the properties of real world networks. In particular, one could venture that the organization of society around small social groups might be a way for large groups of people to form structures that can ensure meritocracy in the local context of a group of peers. More research will be required to answer these questions and help us elucidate the role that networks play in defining the boundaries between meritocracy and topocracy.

## Methods

**Threshold computation for the uniform sharing rule.** We use equation (8) to calculate the total payoffs associated with the *Rockstar* role:

$$\sum_{i=1}^{N} CT_i = C\langle T\rangle N \qquad (12)$$

Dividing the *Rockstar* payoffs ($\Pi_R$) by the total payoffs paid in the entire system ($\Pi = N(N-1)\langle T\rangle$) shows that the fraction of payoffs assigned through the *Rockstar* channel is equal to:

$$\frac{\Pi_R}{\Pi} = \frac{C}{N-1} \qquad (13)$$

Finally, we ask for this ratio to be larger than 1/2 to obtain the condition

$$C > \frac{N-1}{2} \qquad (14)$$

We can solve this condition analytically for a random ER network by assuming a large network ($N \gg 1$) and an average degree larger than one ($\langle k\rangle > 1$). This implies that we can approximate $C$ by the largest term of the sum in eqn. (6). Using the expression for $\ell$ in equation (5) we can re-express (14) as:

$$\frac{N-1}{2} < \sum_{i}^{l} \frac{\langle k\rangle^i}{i} \simeq \frac{\langle k\rangle^l}{l} = \frac{\langle k\rangle^{\ln N/\ln\langle k\rangle}}{\ln N/\ln\langle k\rangle} \qquad (15)$$

Finally, using the change of variable $u = \ln\langle k\rangle/\ln N$ in (15), and approximating $(N-1)/N \simeq 1$ the condition becomes

$$\frac{1}{2} < u, \qquad (16)$$

which is equivalent to: $\langle k\rangle > N^{1/2}$.

**Threshold computation for the percentage sharing rule.** For the sharing rule involving commissions, the total payoff collected by *Rockstars* (when $k(1-\alpha) > 1$) can be approximated by:

$$\Pi_R = N\langle T\rangle \sum_{i=1}^{\ell} \langle k\rangle^i (1-\alpha)^{i-1} \simeq N\langle T\rangle \langle k\rangle^{\ell} (1-\alpha)^{\ell-1}, \qquad (17)$$

The total payoffs paid by the system ($\Pi = N(N-1)\langle T\rangle$) are not changed by the payoff sharing rule. Hence, the fraction of the payoff collected by the *Rockstars* through the meritocratic channel can be obtained similarly than before as:

$$\frac{\Pi_R}{\Pi} = (1-\alpha)^{\frac{\ln N}{\ln\langle k\rangle} - 1}, \qquad (18)$$

which yields to the condition $\langle k\rangle > N^{\frac{\ln(1-\alpha)}{\ln(1-\alpha)/2}}$.

**The statistical meritocracy of networks.** Here, we present an statistical method to estimate the meritocracy $\mathcal{M}$ and *topocracy* $\mathcal{T}$ of the model. Instead of looking at a threshold of $\Pi_R/\Pi$, we define the meritocracy $\mathcal{M}$ as the correlation between an individual's contribution to the network, represented by its talent $T_i$, and the total payoff $\pi_i$ collected by the individual. Topocracy is defined as the correlation between connectivity ($k_i$) and payoff ($\pi_i$) Formally, we define meritocracy in terms of Pearson's correlation as:

$$\mathcal{M} = \mathrm{corr}\left(\vec{T}, \vec{\pi}\right). \qquad (19)$$

By definition, when the network is fully connected, the system is perfectly meritocratic, since in that limit $\pi_i = (N-1)T_i$ and hence $\mathcal{M} = \mathrm{corr}\left[\vec{T}, (N-1)\vec{T}\right] = 1$. In general we can express the total payoff of an individual as the sum of its contributions coming from the creation and distribution of content. Hence, we can rewrite (19) as:

$$\mathcal{M} = \mathrm{corr}\left(\vec{T}, \vec{\pi}_R + \vec{\pi}_M\right). \qquad (20)$$

Next, we decompose the correlation function in its covariance and standard deviation components to obtain:

$$\mathcal{M} = \mathrm{corr}\left(\vec{T}, \vec{\pi}_R + \vec{\pi}_M\right) = \frac{\mathrm{cov}\left(\vec{T}, \vec{\pi}_R\right) + \mathrm{cov}\left(\vec{T}, \vec{\pi}_M\right)}{\sigma_T \sigma_{\pi_R + \pi_M}}. \qquad (21)$$

Finally we use the properties of the variance and covariance, and the fact that $\mathrm{cov}\left(\vec{T}, \overrightarrow{\pi_M}\right) = 0$ to obtain:

$$\mathcal{M} = \frac{C\sigma_T^2}{\sigma_T\sqrt{\sigma_{\pi_R}^2 + \sigma_{\pi_M}^2}} \qquad (22)$$

We can further simplify (22) by noticing that $T$ and $\pi_R$ are uniformly distributed. For this reason, $\sigma_T^2 = 1/12$ and $\sigma_{\pi_R}^2 = C^2/12$. With this, equation (22) simplifies to:

$$\mathcal{M} = \frac{C}{\sqrt{C^2 + 12\sigma_{\pi_M}^2}} \qquad (23)$$

By the same token, we can estimate the *topocracy*, of the system as the correlation between $k^2$ and the total payoff

$$\mathcal{T} = \mathrm{corr}\left(\overrightarrow{k^2}, \vec{\pi}_R + \vec{\pi}_M\right) = \frac{\sigma_{\pi_M}}{\sqrt{C^2/12 + \sigma_{\pi_M}^2}} \qquad (24)$$

Finally, we note that both $\mathcal{M}$ and $\mathcal{T}$ can be evaluated analytically for a sparse ER network by taking into account that in this case

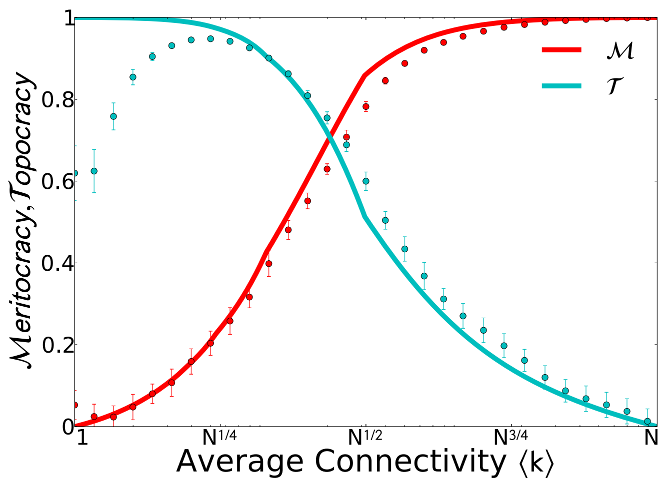


**Figure 5 | Meritocracy (full red line) and topocracy (full blue line), as defined in equations (23) and (24) as a function of the average network connectivity.** The corresponding values obtained by numerical simulation using 500 realizations in a random Erdös-Rényi network with $N = 1000$ nodes are shown for comparison with red and blue dots, respectively.

$$\sigma_{\pi_M} = B^2 \langle k \rangle \left(1 + 6\langle k \rangle + 4\langle k \rangle^2\right). \tag{25}$$

The last two expressions are illustrated in Figure 5, and show that the meritocracy of the system, $\mathcal{M}$, decreases (from 1 to 0) as the network becomes sparse and $\sigma^2_{\pi_M}$ increases, while the opposite is true for the *topocracy*, $\mathcal{T}$, which approaches one for $\sigma^2_{\pi_M} \gg C^2/12$. We note, however, that there is a decrease in *topocracy* at low connectivities that is not accompanied by an increase in meritocracy. This is due to the fact that as $\langle k \rangle \to 1$ the network becomes disconnected, and purchases are only completed in the connected components.

**Generalization to arbitrary prices.** Do prices have an effect on the connectivity threshold separating the meritocratic and topocratic regime of the system that was discussed in Meritocratic and topocratic regimes - general case. To explore this question we assume that each individual sells her content at a price $s_i$. Under this assumption the total payoff collected by *rockstars* is equal to:

$$\Pi_R = \sum_i \pi_{R_i} = C \sum_i T_i s_i \tag{26}$$

whereas the total payoff of the system is given by

$$\Pi = \Pi_R + \Pi_A = \sum_i \sum_{j \neq i} T_i s_i = (N-1) \sum_i T_i s_i. \tag{27}$$

Hence, the threshold condition $\Pi_R/\Pi = C/(N-1)$ is identical than the one found when the prices are equal for all individuals. This means that the threshold separating the meritocratic and topocratic regimes ($k = N^{1/2}$) is valid for an arbitrary vector of prices, and is therefore true even for a system where prices are not in equilibrium. Moreover, we note that the equality $\Pi_R/\Pi = C/(N-1)$ depends only on $C\left(\vec{k}\right)$, which is a structural parameter of the network. Hence, generalizations of these results to alternative network topologies will always be independent of prices and are reduced to determining the relationship between $N$ and $\vec{k}$ that satisfies the condition $C/(N-1) \geq 1/2$.


1. Granovetter, M. Economic action and social structure: the problem of embeddedness. *Am. J. Sociol.* **91**, 481–510 (1985).
2. Uzzi, B. The sources and consequences of embeddedness for the economic performance of organizations: The network effect. *Am. Sociol. Rev.* **61**, 674–698 (1996).
3. Uzzi, B. Social structure and competition in interfirm networks: The paradox of embeddedness. *Admin. Sci. Q.* **42**, 35–67 (1997).
4. Biglaiser, G. Middlemen as experts. *RJE* **24**, 212–223 (1993).
5. Li, Y. Money and middlemen in an economy with private information. *Econ. Inq.* **90**, 1–12 (1999).
6. Akerlof, G. A. The market for lemons: Quality uncertainty and the market mechanism. *Q. J. Econ.* **84**, 488–500 (1970).
7. Rubinstein, A. & Wolinsky, A. Middlemen. *Q. J. Econ.* **102**, 581–593 (1987).
8. Wong, Y.-Y. & Wright, R. Buyers, Sellers and Middlemen: Variations on Search-Theoretic Themes. *Working Paper*, http://www.minneapolisfed.org/research/wp/wp691.pdf, *10/11/2013*, (2011).
9. Burt Ronald, S. *Structural holes: The social structure of competition*. (Harvard University Press, 2009).
10. Burt Ronald, S. *Structural holes and good ideas.* American journal of sociology **110**, 349–399 (2004).
11. Jackson, M. O. & Wolinsky, A. A strategic model of social and economic networks. *JET* **71**, 44–74 (1996).
12. Jackson, M. O. *Social and economic networks* (Princeton University Press, 2010).
13. Easley, D. & Kleinberg, J. *Networks, Crowds, and Markets: Reasoning about a Highly Connected World* (Cambridge University Press, 2010).
14. Galeotti, A., Goyal, S., Jackson, M. O., Vega-Redondo, F. & Yariv, L. Network games. *The Review of Economic Studies* **77**, 218–244 (2010).
15. Goyal, S. & Vega-Redondo, F. Structural holes in social networks. *JET* **137**, 460–492 (2007).
16. Hojman, D. A. & Szeidl, A. Core and periphery in networks. *JET* **139**, 295–309 (2008).
17. Kets, W., Iyengar, G., Sethi, R. & Bowles, S. Inequality and network structure. *GEB* **73**, 215–226 (2011).
18. Dunbar, R. I. The social brain hypothesis. *brain* **9**, 10. (1998).
19. Barabasi, A.-L. & Albert, R. Emergence of scaling in random networks. *Science*, **286**, 50 (1999).
20. Yule, G. U. A mathematical theory of evolution, based on the conclusions of Dr. JC Willis, FRS. *Phil. Trans.* Series B, **213**, 21–87 (1925).
21. Price, D. J. de S. A general theory of bibliometric and other cumulative advantage processes. *J. Amer. Soc. Inform. Sci.* **27**, 292–306 (1976).
22. Merton, R. K. The Matthew effect in science. *Science* **159**, 56–63 (1968).
23. Simon, H. A. On a class of skew distribution function. *Biometrika* **42**, 425–440 (1955).
24. Simon, H. A. Designing organizations for an information-rich world *Computers, communications, and the public interest* Greenberger, M. (ed) 37–72 (Baltimore, 1971).
25. Gleick, J. *The information: A history, a theory, a flood* (HarperCollins UK, 2011).
26. Erdös, P. & Rényi, A. On the evolution of random graphs. *Magyar Tud. Akad. Mat. Kutat Int. Kol.* **5**, 17–61 (1960).
27. Barthelemy, M. Betweenness centrality in large complex networks. *Eur. Phys. J. B* **38**, 163–168 (2004).
28. Stiglitz, J. E. *The price of inequality: How today's divided society endangers our future* (W.W. Norton & Co., 2012).
29. Marx, K. *Critique of the Gotha program* (Wildside Press LLC, 2008).
30. Friedman, M. & Friedman, R. [Episode 5: Created Equal, PBS TV Series] *Free to choose*(Harcourt Press, 1980).
31. Coleman, J. S. Social capital in the creation of human capital. *Am. J. Sociol.* S95–S120 (1988).
32. Rawls, J. *A Theory of Justice* (Cambridge: Belknap of Harvard UP, 1999).
33. Rawls, J. *Justice as Fairness: A Restatement* (Cambridge: Belknap of Harvard UP, 2001).
34. Santos, F. C., Santos, M. D. & Pacheco, J. M. Social diversity promotes the emergence of cooperation in public goods games. *Nature* **7201**, 213–216 (2008).
35. Perc, M. & Szolnoki, A. Social diversity and promotion of cooperation in the spatial prisoner's dilemma game. *PRE* **77**, 1, 011904 (2008).
36. Gómez-Gardeñes, J., Campillo, M., Floría, L. M. & Moreno, Y. Dynamical organization of cooperation in complex topologies. *PRL* **98**, 10, 108103 (2007).



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## Author contributions

J.B., F.B. and C.H. devised the model and did the analytical calculations. J.B. and C.H. prepared the figures. J.B., F.B., C.R.-S. and C.H. discussed the results and wrote the manuscript.

## Additional information

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